

Quantum Look at two Common Logics: the Logic of Primitive Thinking and the Logic of Everyday Human Reasoning

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Based on ideas of quantum theory of open systems and psychological dual system theory we propose two novel versions of Non-Boolean logic. The first version can be interpreted in our opinion as simplified description of primitive (mythological) thinking and the second one as the toy model of everyday human reasoning in which aside from logical deduction, heuristic elements and beliefs also play the considerable role. Several arguments in favor of the interpretations proposed are adduced and discussed in the paper as well.

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I. INTRODUCTION

It is well known that the Boolean propositional logic which is consistent mathematical presentation of the classical Aristotelian logic has applications of two kinds. On the one hand it correctly takes into account deductive elements of the everyday human reasoning and on the other hand this logic can be considered as the relevant framework of common scientific language both in exact sciences and humanities. For example referring to the classical physics it may be argued that the result of any experiment realized in this area can be described by the Boolean logic in a consistent manner. However in the situations when distinct features of the objects under study strongly correlate between themselves and moreover may be incompatible the application of standard Boolean logic may lead to errors. Just such case has place for example in quantum mechanics. Actually, as firstly G. Birkhoff and J. von Neumann revealed [1] in quantum mechanics, in view of existence of certain non-commuting observables relating to the same particle such as coordinate and momentum or the different components of the spin, the laws of the Boolean logic (in particular the distributive law) have been failed and their generalization i.e. quantum logic is desirable. The similar situation may arise also in everyday life when various reasons, emotions and beliefs governing the behavior of concrete person begin to contradict each other. In this connection it should be noted that still in the early days of quantum theory N. Bohr, when he formulated the Complementarity Principle, (which just maintains the presence in physics certain supplementary properties of the microscopic objects) underlined more than once essential analogy existing between atomic processes and such mental phenomena as thoughts, sentiments and acts of decision making. It is remarkable that modern cognitive psychology using the data of a large number of experiments came to the similar conclusions as well. Really according to the psychologi-

cal dual system theory all basic cognitive processes such as attention, memory, learning and so on are connected with certain dual systems and dual mechanisms. In the present paper we will interested in only human reasoning mechanisms where two primary dual systems of interest can be specified. The first of them (we will term it below as deductive reasoning system) is rational, sequential and consistent but acts relatively slow and its resources are limited by the capacity of human working memory. The second one (we will term it as heuristic reasoning system) is intuitive, rapid and automatic but its activity may be biased to a large degree by emotions and old unconscious ideas. In addition a hidden interaction exists between these two cognitive systems which as a rule is not aware by the reasoning person. The main goal of present paper is to demonstrate how by using some ideas, concepts and technical tools of quantum theory of open systems (QTOS) one can describe the dual nature of human reasoning in the framework of consistent logical theory that generalizes and modifies the rules of ordinary Boolean logic.

The paper is organized as follows. In Section.2 following the previous author paper [2] we briefly outline main ideas of the approach proposed using the simple and instructive example of probabilistic Boolean logic (PBL) that describes the situations when in the absence of complete information about the surrounding objects and events all human judgments acquire inevitably probabilistic nature. To take into account this crucial point it is convenient, instead of usual Boolean functions of discrete variables which take only two values unit and zero, to associate with every plausible proposition (PP) certain diagonal representative density matrix (RDM) of some auxiliary two state quantum system, whose elements define the plausibility of corresponding proposition. The essential and novel element of the approach proposed is the universal and constructive algorithm (based on QTOS) which allows one to define all logical connectives between plausible propositions using the powerful and effective method of positive valued (PV) quantum operations. It is worth noting that this method will be used continually throughout the paper for the construc-

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tion of various kinds of logics. Also we study here another important problem, namely : how the possible logical correlations between various PP may be taken into account in the approach proposed. Further in the Section.3 in order to describe the dual nature of human reasoning we have extended our approach to the arbitrary 2×2 non-diagonal density matrices that will represent in this case the generalized propositions (GP). We assume that non-diagonal elements of the RDMs of such propositions relate to heuristic (believable) elements of human reasoning and therefore by means of them one can define the believability of the corresponding propositions. The main problem of the approach proposed is: whether one may specify the set of logical connectives for similar generalized propositions and if the answer is yes in what way. To answer this question we will use again the approved method of PV quantum operations. It turns out however that in general case (beyond the usual PBL) it is necessary to impose certain restrictions either on the form of admissible quantum operations or on the form of input GP. In this connection we proposed two possible alternatives to construct the consistent Non-Boolean (NB) logic operating with such GP. The NB logic of the first kind aside from negation includes only the single pair of two place connectives which are similar to the pair of relations: equivalence-non-equivalence in the standard Boolean logic. At first sight such logic looks much poorer than standard Boolean logic and we will term it as 'atomic' logic or prime logic. It turns out however that, starting from GP and using only these two connectives, one may construct any prescribed diagonal PP and after that to handle with these PP according to the known rules of PBL. Thus PBL appears now as secondary or 'molecular' logic with respect to prime or 'atomic' NB logic of the first kind.

We argue that described version of Non-Boolean logic of the first kind (prime logic) can be considered as simplified version of the logic of primitive (or mythological) thinking. In favor of such interpretation we adduce several cogent in our opinion arguments. Finally in the Section.4. we study the NB logic of the second kind that assumes the existence of special primordial correlations between deductive and heuristic components of generalized propositions. In this case using the formalism of PV quantum operations one can specify all the same logical connectives between GP including implication as in usual PBL. It turns out that a number of known from everyday life curious psychological phenomena such as belief-bias effect may be explained in the framework of this logic by natural way. Now let us go to the concrete presentation of announced results.

II. PRELIMINARIES

In this section we briefly remind some results of our previous paper [2] in which based on ideas of quantum theory of open systems the formalism of probabilistic

Boolean logic (PBL) was developed. To this end in view, instead of standard Boolean variables with two values 1 and 0 that represent true and false propositions respectively one has to consider so called plausible propositions (PP), whose truth or falsity are determined only with certain probability. We propose to represent such propositions by means of 2×2 diagonal matrices with positive elements the sum of which is equal to unit. Every such representative matrix of PP may be associated with the density matrix of certain two state quantum system. Therefore, if it does not lead to confusion, we will often identify the propositions in PBL with their representative density matrices (RDMs).

In explicit form the RDM of arbitrary PP a looks as $\rho(a) = \begin{pmatrix} p_a & 0 \\ 0 & 1 - p_a \end{pmatrix}$ where p_a is a probability for proposition a to be true (index a of the proposition we will often omit later). It is convenient also to define the plausibility of the proposition a as $P(a) = 2p_a - 1$. Obviously that plausibility takes its values in the interval $[-1, 1]$. It turns out the approach proposed allows one to express all logical connectives between PP by universal way as certain positive valued (PV) quantum operations under their RDMs. Referring the reader for the details of this approach to [2] let us merely demonstrate here how this approach works using several concrete examples. So, if someone wants to obtain the RDM of the negation of PP $a = \begin{pmatrix} p & 0 \\ 0 & 1 - p \end{pmatrix}$ that is (not a) she(or he) must make the next quantum operation: $\rho(nota) = G_{not}\rho(a)G_{not}^T$ with the following 2×2 matrix $G_{not} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and G^T is, as usually, the matrix transposed to matrix G . It turns out that any two place connective in PBL (aRb) may be expressed in similar manner as well, namely:

$$(aRb) = G_R [\rho(a) \otimes \rho(b)] G_R^T \quad (1)$$

where $\rho(a) \otimes \rho(b)$ is standard tensor product of RDMs of propositions a and b and G_R is a 2×4 matrix (we will term it below as admissible matrix for connective R) which has two defining properties: 1) every element of G_R is equal 0 or 1 and 2) in each column of matrix G_R the only element is equal to one and all the rest are equal to zero. Let us now present (without proof) the concrete form of matrices G_R for basic two place connectives, namely : (a and b), (a or b), ($a \implies b$). As shown in [2] they have the next form:

$$G_{(a \text{ and } b)} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix}, \quad (2)$$

$$G_{(a \text{ or } b)} = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (3)$$

$$G_{(a \implies b)} = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}. \quad (4)$$

For example for the implication ($a \implies b$) one can easily

writes down the RDM as:

$$\rho(a \Rightarrow b) = \begin{pmatrix} 1-p+pq & 0 \\ 0 & p(1-q) \end{pmatrix}. \quad (5)$$

In Eq. (5) we mean the next notation: $a = \begin{pmatrix} p & 0 \\ 0 & 1-p \end{pmatrix}$, and $b = \begin{pmatrix} q & 0 \\ 0 & 1-q \end{pmatrix}$. Note that this relation coincides with standard Boolean expression for the implication in the case when probabilities p and q may take only two values 0 and 1. Let us make now the generalization of the approach proposed on the important case when initial PPs a and b are not independent propositions or, in other words, they possess some logical correlations. In such situation the best way to take these correlations into account is to avail of the direct analogy between our approach and the theory of composite correlated quantum systems. To realize this analogy one must take as a starting point, instead of tensor production of two initial PPs a and b the more general positive

diagonal matrix of the form: $\rho(a, b) = \begin{pmatrix} p_1 & 0 & 0 & 0 \\ 0 & p_2 & 0 & 0 \\ 0 & 0 & p_3 & 0 \\ 0 & 0 & 0 & p_4 \end{pmatrix}$

with additional normalization condition: $\sum_{i=1}^4 p_i = 1$. Furthermore we naturally assume that this joint matrix of correlated propositions $\rho(a, b)$ is connected with matrices of partial propositions that is RDM of a and RDM of b by the standard relations: $a = \begin{pmatrix} p_1 + p_2 & 0 \\ 0 & p_3 + p_4 \end{pmatrix}$, $b = \begin{pmatrix} p_1 + p_3 & 0 \\ 0 & p_2 + p_4 \end{pmatrix}$. Now let us suppose again that all logical connectives for correlated PP may be obtained by the approved method that is by means of the PV quantum operations. Here we restrict ourselves only with the case of implication between correlated propositions a and b . Let us introduce the following notation $p = p_1 + p_2$, $q = p_1 + p_3$ and $C = p_1 p_4 - p_2 p_3$

It is easy to see that ,using this notation, the matrix $\rho(a, b)$ can be rewritten in the form: $\rho(a, b) = \begin{pmatrix} pq+C & 0 & 0 & 0 \\ 0 & p(1-q)-C & 0 & 0 \\ 0 & 0 & q(1-p)-C & 0 \\ 0 & 0 & 0 & (1-p)(1-q)+C \end{pmatrix}$.

Now by applying the quantum operation $G_{(a \Rightarrow b)} = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$ to the $\rho(a, b)$ one may obtain the required result for RDM of implication, namely:

$$\rho(a, b) = \begin{pmatrix} 1-p+pq+C & 0 \\ 0 & p-pq-C \end{pmatrix}. \quad (6)$$

Note that the variable C is just a measure of logical correlations between propositions a and b . In this connection we will termed it as the context variable.

Let us consider now the simple example with three concrete initial RDMs of composite correlated propositions, namely :

$$\begin{aligned} 1) \rho_1(a, b) &= \begin{pmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 \end{pmatrix}, \\ 2) \rho_2(a, b) &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \text{ and} \\ 3) \rho_3 &= \begin{pmatrix} \frac{1}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{4} & 0 & 0 \\ 0 & 0 & \frac{1}{4} & 0 \\ 0 & 0 & 0 & \frac{1}{4} \end{pmatrix}. \end{aligned}$$

It is easy to see that in all cases the RDMs for partial propositions a and b coincide and represent the same ambiguous proposition: $\rho_i(a) = \rho_i(b) = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$ ($i = 1, 2, 3$). However the expressions for implication in these situations differ significantly. Really using the general rule Eq. (1) stated above and its special case in the form of Eq. (6) one may easily obtain:

$(a \Rightarrow b)_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, that is the true proposition, while $(a \Rightarrow b)_2 = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$ is the same ambiguous proposition and $(a \Rightarrow b)_3 = \begin{pmatrix} \frac{3}{4} & 0 \\ 0 & \frac{1}{4} \end{pmatrix}$. Note that only the last case

with composite matrix ρ_3 corresponds to the case of the independent partial propositions. This simple but instructive example clearly demonstrates the considerable influence of logical correlations on the results of inference process. In this point we have finished our brief review of PBL. It should be noted in conclusion that in the case of PBL considered in this Section were taken into account only propositions that may be specified by the single defining characteristic , namely plausibility. Now we pass to the study of Non-Boolean (NB) logics that would operate with the propositions of more general kind .These propositions can be determined already by two defining features which reflect dual nature of the objects (or subjects) connecting with corresponding propositions . We believe that similar logics may be more appropriate for the description (at least in simplified form) of essential peculiarities of such common logics as the logic of primitive (mythological) thinking and the logic of everyday human reasoning.

III. NON-BOOLEAN LOGIC OF THE FIRST KIND AS THE SIMPLIFIED DESCRIPTION OF PRIMITIVE (MYTHOLOGICAL) THINKING

In the beginning of this Section we would like to discuss the following primary question namely: to what degree the approach stated above may be extended on the propositions of more general form .To this end in view let us consider instead of exclusively diagonal RDM representing a certain PP the more general class of 2×2

non-diagonal positive valued matrices with unit trace and examine them as certain RDMs of generalized propositions (GP). Let us suppose that an arbitrary proposition of this class a can be written in the next form $\rho(a) = \begin{pmatrix} p & z \\ z^* & 1-p \end{pmatrix}$, where z is some complex number. Let us define also the negation of such proposition, that is ($not\ a$) by natural way as $\rho(not\ a) = \begin{pmatrix} 1-p & z^* \\ z & p \end{pmatrix}$. Furthermore let us require that the RDMs of the opposite propositions a and $not\ a$ commute with each other. The sense of this restriction (having in mind the obvious analogy with quantum theory) is quite clear and does not need in additional comments. It is easy to see that this restriction may be reduced to the simple relation $z = i\alpha$ or, in other words, non-diagonal elements of the RDM in the approach proposed have to be pure imaginary. Thus the class of generalized RDMs which we will consider in the present Section contains all 2×2 positive valued matrices of the form:

$$\rho(a) = \begin{pmatrix} p & i\alpha \\ -i\alpha & 1-p \end{pmatrix}. \quad (7)$$

As to formal interpretation of these GPs we assume as before that their diagonal elements determine the plausibility of corresponding GP according to the above mentioned rule: $P(a) = 2p - 1$ whereas their non-diagonal elements define its believability, that is the readiness of a subject to accept the corresponding proposition as valid by virtue of some irrational and unconscious reasons or, in other words, on the strength of certain beliefs. Thus the similar GPs combine together the objective and subjective reasons which stimulate a person to accept the concrete proposition as valid. It is naturally to define the believability of concrete GP a with RDM Eq. (9) as $B(a) = -2\alpha$. The justification of such definition became clear if we write down this GP by standard way using the Bloch sphere representation, namely:

$$\rho(a) = \begin{pmatrix} \frac{1+P_z}{2} & \frac{P_x-iP_y}{2} \\ \frac{P_x+iP_y}{2} & \frac{1-P_z}{2} \end{pmatrix}. \quad (8)$$

Note that for GPs which we consider here, the component P_x of the Bloch vector \vec{P} is always equal to zero. Now if one compares the representation Eq. (8) with expression Eq. (7) she(he) can see that the plausibility and believability of the same GP being expressed in terms of the Bloch vector components look similarly. Furthermore it should be noted that relation $P_z^2 + P_y^2 \leq 1$ is obviously holds for any GP and this fact distinctly reflects the competition existing between two features of the same proposition. It should be noted also that our assumption is in a full agreement with modern psychological dual system theory [3] that asserts the presence in brain two complementary cognitive systems: (deductive and heuristic), which both are responsible for human reasoning, although often compete with each other. Now in order to formulate the consistent NB logic with GPs stated above

we have to define all possible connectives related to them. It turns out however that the direct extension of the admissible quantum operation method in order to obtain required two place logical operations for GPs is impossible (because the automatic satisfaction of normalization condition, that was guaranteed for PPs in PBL, is not always holds now). However there are at least two possible ways to get round this formal obstacle. In this Section we consider the first of them. To this end we retain as admissible operations only two concrete 2×4 matrices, namely: $G_\Delta = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$ and $G_{\bar{\Delta}} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$. It is easy to see directly that these operations transform the tensor product of two propositions $a \otimes b$ to the RDM that belongs to the required class of GP. Really, let $a = \begin{pmatrix} p & i\alpha \\ -i\alpha & 1-p \end{pmatrix}$ and $b = \begin{pmatrix} q & i\beta \\ -i\beta & 1-q \end{pmatrix}$ are two GP, then it is easy to obtain that:

$$G_\Delta(a \otimes b) G_\Delta^T = \begin{pmatrix} A & i\gamma \\ -i\gamma & 1-A \end{pmatrix} \equiv (a\Delta b). \quad (9)$$

where $A = p + q - 2pq + 2\alpha\beta$ and $\gamma = \alpha(1-2q) + \beta(1-2p)$. In the same way one can find that

$$G_{\bar{\Delta}}(a \otimes b) G_{\bar{\Delta}}^T = \begin{pmatrix} 1-A & -i\gamma \\ i\gamma & A \end{pmatrix} = \quad (10) \\ = not(a\Delta b) \equiv (a\bar{\Delta}b).$$

In the case when x -component of the Bloch vector is identically equal to zero it is convenient to introduce the complex Bloch vector as follows: $P = P_z - iP_y$. Comparing this definition with the above mentioned Bloch sphere representation of density matrix one can state the simple relation connecting the complex vector R of the proposition $(a\Delta b)$ with corresponding vectors P and Q of the GP a and b respectively, namely:

$$R = -PQ. \quad (11)$$

So, we found that the NB logic of the first kind aside from negation contains in addition only the single pair of two place connectives, namely Δ and $\bar{\Delta}$. At first sight such logic is much poorer than standard Boolean logic and therefore one may call it as 'atomic logic' or prime logic. Nevertheless it should be noted that, having in hands only these two connectives and starting from the set of GPs, one can easily obtain any PP belonging to PBL with prescribed diagonal elements. After making that, one may operate with them already according to the rules of standard Boolean logic. Let us demonstrate the validity of this statement. To this end let us take two opposite GP of the special form: $a = \begin{pmatrix} \frac{1}{2} & i\alpha \\ -i\alpha & \frac{1}{2} \end{pmatrix}$ and ($not\ a$) = $\begin{pmatrix} \frac{1}{2} & -i\alpha \\ i\alpha & \frac{1}{2} \end{pmatrix}$. If one applies the operation Δ to them the obtained result looks as:

$$(a\Delta not\ a) = \begin{pmatrix} \frac{1}{2} - 2\alpha^2 & 0 \\ 0 & \frac{1}{2} + 2\alpha^2 \end{pmatrix}. \quad (12)$$

Thus we see that any proposition a of PBL with $p_a \leq \frac{1}{2}$ may be expressed by similar way. As for propositions of PBL with $p_a \geq \frac{1}{2}$ they can be expressed by means of operation $\bar{\Delta}$ in the same manner. Thus the PBL plays now the role of secondary or 'molecular' logic with respect to prime or 'atomic' NB logic of the first kind. Note in addition that the GP of the form : $a = \begin{pmatrix} \frac{1}{2} & i\alpha \\ -i\alpha & \frac{1}{2} \end{pmatrix}$ whose plausibility is equal to zero can be considered as the simplest example of so-called intuitive judgments. In this place we refer the reader to the inspiring book [4] in which distinction between discursive and intuitive judgments have been deeply analyzed. Now let us pass to the most intriguing question, relating to the NB logic of the first kind stated above, namely: whether the logic proposed above has any actual applications in everyday life and (or) in science? We believe that the answer is yes and are going to argue that logic of this kind could be appropriate for example as simplified description of primitive (mythological) thinking. The author is aware that without being an expert in the area of anthropology or mythology he is unable to disclose this topic in proper depth and moreover to prove the above assertion exactly. Nevertheless let me adduce several arguments in favor of the hypothesis proposed above. In the beginning let us remind some basic facts relating to the primitive thinking (PT), or the thinking of savages (all these and the other concrete facts the reader can derive in detail for example from classical book by C. Levi-Strauss) [5]. The major fact on which we base is that there are a set of elementary and at the same time fundamental units of PT, so-called binary oppositions, such as : top-bottom, right-left, birth-death, male-female and so on. Another important regularity determines the basic principle governing the primitive thinking, that is, its goals and mental tools which are used to achieve these goals . It turns out that PT while it operates with binary oppositions does not seek to avoid any contradictions, that is typical for the "normal" logic, but rather tends to find and then to reconstruct all kinds of possible intermediate links existing between two antagonistic terms in concrete binary opposition. To this end in view there are two special personages in myth, namely: the Culture Hero and the Trixter, (for example in Greek mythology Hermes plays the role of such Trixter) which realize this important function. Furthermore certain totem animals for example the Raven and the Coyote in Indian myths sometimes play the part of Culture hero as well. The basic tool using in PT that allows one to achieve the desired goal is so-called 'mediation' –the special mental or workable operation which is able to reduce the distinction between two antagonistic terms in binary oppositions. For example the Raven and the Coyote as the animals- scavengers symbolically realize the mediation function between the world of the living beings and the world of the dead . Now let us demonstrate how in the NB logic of the first kind described above its two basic operations may be considered as certain analogues of the mediation operation in myth.

To this end, in order to define the degree of difference between two distinct GPs, we will use well-known definition of the distance between two mixed quantum states ρ_1 and ρ_2 , namely $D(\rho_1, \rho_2) = \frac{1}{2}Sp|\rho_1 - \rho_2|$ [6]. It is easy to see that in the case of two dimensional Hilbert space and in the Bloch sphere representation the above expression takes the simple form: $D(\rho_1, \rho_2) = \frac{1}{2}|\vec{P}_1 - \vec{P}_2|$ where \vec{P}_1 and \vec{P}_2 are corresponding Bloch vectors of the states.

It is obvious that maximum distance is realized between opposite propositions or , (using the PT language) between antagonistic members of binary opposition. Now if one applies the basic two place operation Δ to the pair of opposite propositions a with complex vector P and (*nota*) with complex vector $-P$ she(he) obtains the new proposition with the complex vector $R = P^2$. It is easy to see that the distance between the opposite propositions with complex vectors P^2 and $-P^2$ is less than the distance between original pair of opposite propositions. This fact confirms that logical connective Δ possesses the characteristic property of operation 'mediation' in PT. Now we are going to adduce another forcible argument in favor of the doubtless connection between NB logic of the first kind considered above and the logic of PT. Let us remind that according to the known ethnologist Levy-Bruhl [7] (who was the first European researcher of primitive and magical thinking PT is governed to a great extent by the so-called law of participation that claims the universal links existing between various things and events in the world. In particular just this law allows the shaman, who fell into trans, to perform a wide variety of magical acts and transformations with surrounding objects or people using their mutual contiguity and (or) similarity. It is interesting to note that this feature of primitive thinking afterwards was adopted by art and literature (especially poetry) in the form of extensive use of such specific tropes as metaphor and metonymy. In the NB logic of the first kind proposed above this feature of PT may be simulated by additional logic operations, namely by logic rotations, that have no analogues in the standard Boolean logic. Really, it is possible to define one-parameter continuous group of logic rotations (in the plane $P_z - P_y$) according to the usual mathematical rule, namely: let GP a is represented by the complex vector P then GP \tilde{a} obtained by rotation of a at an angle Φ has complex vector \tilde{P} with components \tilde{P}_y and \tilde{P}_z so that $\tilde{P}_y = P_y \cos \Phi + P_z \sin \Phi$ and $\tilde{P}_z = P_z \cos \Phi - P_y \sin \Phi$. Clearly that in this group of logical transformations the negation of the proposition a coincides with its rotation at angle π and, furthermore, if one rotates GP a at angle Φ_1 and the other GP b at angle Φ_2 then the GP $(a\Delta b)$ would be rotated at angle $(\Phi_1 + \Phi_2)$. Thus this continuous group of logical rotations let one to transform any proposition into the arbitrary other (with the same modulus certainly). In this point we are obliged to finish our brief discussion of the relationship existing between primitive thinking and the NB logic of the first kind although it

is clear that this complex and intriguing topic deserves much more detail study.

IV. NON-BOOLEAN LOGIC OF THE SECOND KIND AS THE TOY MODEL OF EVERYDAY HUMAN REASONING

As we see in the previous Section in the case of Non-Boolean logic of the first kind it is possible to define aside from negation only the single pair of two- place logical operations Δ and $\bar{\Delta}$. The application of the rest admissible quantum operations to the tensor product of two input GP results in the disturbance of normalization condition for output RDM. This obstacle may be formally removed if one assume the existence of certain primordial correlations between plausible and believable components of the GPs. Let us explain more detail what we have in mind. Suppose that instead of tensor product of the two input GP $a \otimes b$ (that means their logical independence) we take as input more general composite proposition $\rho(a, b)$, the RDM of which has the next form:

$$\rho(a, b) = \begin{pmatrix} p_1 & i\alpha & i\beta & 0 \\ -i\alpha & p_2 & 0 & i\gamma \\ -i\beta & 0 & p_3 & i\delta \\ 0 & -i\gamma & -i\delta & p_4 \end{pmatrix}. \quad (13)$$

Using the obvious analogy with theory of composite quantum systems one can determine now the partial RDM of propositions a and b as $a = \begin{pmatrix} p_1 + p_2 & i(\beta + \gamma) \\ -i(\beta + \gamma) & p_3 + p_4 \end{pmatrix}$ and $b = \begin{pmatrix} p_1 + p_3 & i(\alpha + \delta) \\ -i(\alpha + \delta) & p_2 + p_4 \end{pmatrix}$ respectively. It is clear that input composite proposition $\rho(a, b)$ Eq. (13) corresponds to the case of two correlated partial GP a and b and the degree and nature of their correlation is uniquely determined by elements of composite matrix $\rho(a, b)$. It should be emphasized that construction of the input composite proposition $\rho(a, b)$ in the form Eq. (13) has the essential advantage over simple tensor product from formal point of view since it allows one to define all 16 two place logical connectives consistently by approved method of PV quantum operations. Significantly that all admissible matrices G_R in this NB logic of the second kind have the same form as in the case of PBL (see Eq. (1)) and corresponding two place connectives can be obtained according to the similar rule:

$$(aRb) = G_R \rho(a, b) G_R^T. \quad (14)$$

Let us present here (without details) the expressions for the two place connectives (*and*), (*or*) in the NB logic of the second kind, obtained by means of Eq. (14):

$$(a \text{ and } b) = \begin{pmatrix} p_1 & i(\alpha + \beta) \\ -i(\alpha + \beta) & p_2 + p_3 + p_4 \end{pmatrix}, \quad (15)$$

$$(a \text{ or } b) = \begin{pmatrix} p_1 + p_2 + p_3 & i(\gamma + \delta) \\ -i(\gamma + \delta) & p_4 \end{pmatrix} \quad (16)$$

Now as in the previous section, the important question arises about the possibility of concrete realizations of the NB logic of the second kind.

In this connection we are going to bring several arguments in favor of hypothesis that above version of NB logic can be considered as simplified model of everyday human reasoning. To this end in view let us remind some basic facts relating to the psychological dual system theory of human cognition (see for example [3] for the comprehensive exposition). As numerous psychological experiments and observations had unambiguously demonstrated there are two distinct types of mental processes that are responsible for human reasoning: 1) cognitive processes of the first type that are rapid, automatic and intuitive and 2) the processes of the second type that are slow, sequent and rational. Cognitive system which is responsible for the processes of the first type is termed as heuristic (or intuitive) while the second one is termed as deductive (or analytical). Clearly these two systems may sometimes come into conflict and compete with each other. Therefore human reasoning is often biased and subjected to various fallacies. In particular the reasoning person often overestimates the cogence of the arguments that lead to the believable and expected conclusions and underestimates those arguments which lead to the conclusions contradictory to her (his) beliefs. This widespread psychological phenomenon is known as belief-bias effect. Now the natural question arises: is it possible to describe this human hybrid intuitive-deductive thinking in the framework of any consistent logical theory? In this Section we propose brief outline of possible theory based on the NB logic of the second kind that was exposed above. Note that we present here only the little fragment of this theory which let one to understand belief-bias effect in the framework of closed logical point of view. To this end let us examine the expression for implication connecting two correlated GP. According to the general rule Eq. (14) the RDM of the implication can be represented in the form:

$$(a \implies b) = G_{imp} \rho(a, b) G_{imp}^T = \begin{pmatrix} p_1 + p_3 + p_4 & i(\alpha - \gamma) \\ -i(\alpha - \gamma) & p_2 \end{pmatrix}. \quad (17)$$

where the matrix $G_{imp} = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$.

We see that the believability of the GP $(a \implies b)$ as it follows from Eq. (17) is equal to $2(\gamma - \alpha)$, and it does not connect by simple way with corresponding believabilities of partial propositions a and b . Let us now impose another additional restriction on the phases $\alpha, \beta, \gamma, \delta$ of the matrix $\rho(a, b)$ which looks enough naturally, namely, we require that in simple version of NB logic of the second kind the known De Morgan dual formulas for all logical propositions, namely: *not* ($a \text{ and } b$) = (*not* a) *or* (*not* b) and *not* ($a \text{ or } b$) = (*not* a) *and* (*not* b) hold. This is just the case what we have in mind when we are speaking about the toy model of everyday human reasoning. Really, it is easy to see that the restriction

above immediately leads to the next condition imposed on the phases of input matrix $\rho(a, b)$ of Eq. (13), namely: $\alpha + \beta = \gamma + \delta = \text{const.}$ Since this constant must be identical for all propositions in the toy model of NB logic of the second kind we can without loss of generality set it equal to zero and then it turns out that all non-diagonal elements in implication (and in other connectives as well) can be expressed by means of the non-diagonal elements of partial propositions. Comparing the believability of the implication $(a \implies b)$ with the believability of it consequence b that is equal to $-2(\alpha + \delta) = 2(\gamma - \alpha)$ we immediately come to the desired conclusion, namely, in the toy version of NB logic of the second kind the values of believabilities of the propositions $(a \implies b)$ and b completely coincide (and both are equal to $2(\gamma - \alpha)$). Strictly speaking just this conclusion gives us the good reason to believe that NB logic of the second kind has essential common features with everyday human logic in which (as was shown in numerous experiments) the believability of conclusions stimulates the subjects more positively evaluate the correctness of the process of their inference and to pay less attention to their logical rigor (belief-bias effect). It is necessary to mention also that as a matter of fact the whole subjective evaluation of the validity of

the given proposition a simultaneously should take into account both the plausibility of this proposition and its believability as well. Therefore the appropriate expression for the integral validity V of the GP a should be actually look as follows:

$$V(a) = \alpha P(a) + (1 - \alpha) B(a). \quad (18)$$

(where the coefficient α reflects the mental nature of reasoning person. It remains to point out that by application of the results of the approach proposed in this Section one can evaluate quantitatively both believability and plausibility of various GPs in complex logical situations relating to everyday human reasoning. However this amusing task is already beyond the scope of the present paper.

In the conclusion we should like to emphasize only that all guesses and conjectures about proposed interpretations of two NB logics expressed in this paper must be taken with a grain of salt, that is as plausible but preliminary hypotheses. Obviously, the rigorous confirmation of all results obtained here will require more painstaking extra work in the future.

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- [1] G. Birkhoff, J. von Neumann, Ann.Math.,37,823, (1936)
 - [2] E.D. Vol, Int.J. of Theor. Phys.,52,514,(2013)
 - [3] Jonathan Evans,Keith Frankish, In Two Minds: Dual Processes and Beyond,Oxford University Press,(2009)
 - [4] E.L. Feinberg, Zwei Kulturen, Springer-Verlag,(1998)
 - [5] C. Levi-Strauss, La Pensee Sauvage, Paris, (1962)
 - [6] M.A. Nielsen, I.L. Chuang, Quantum Computation and Quantum Information, Cambridge University Press, (2001)
 - [7] L. Levy-Bruhl, La Mythologie Primitive, Paris, (1935)